Geometric Sequence

- **A *geometric sequence* is formed by multiplying each term after the first by a constant called the *common ratio (r).***
- **Ex: 4, 12, 36, 108, …**
- ***r* can be positive or negative.**
- **The common ratio can be found by dividing any term by the term before it.**

Ex: What is *r* in each sequence:

1, 4, 16, 64, 256 $r = 4$

24, 36, 54, 81, 121.5 $r = \frac{36}{24} = \frac{3}{2}$

2000, 500, 125, 31.25 $r = \frac{500}{2000} = \frac{1}{4}$

Ex: Find the next three terms: 2, -4, 8, …

$-16, 32, -64$

- **The general term of a geometric sequence is:**

$$t_n = t_1 r^{n-1}$$

t_n = General term

t_1 = First term

r = common ratio

n = number of terms

Examples:

1. **Given the sequence 6, 18, 54, …, find t_{11} (Find the 11th term)**

$t_n = t_1 r^{n-1}$

$t_{11} = 6(3)^{11-1}$

$t_{11} = 6(3)^{10}$

$\boxed{t_{11} = 354\,294}$

$t_1 = 6$

$n = 11$

$r = 3$

2. **Create a geometric sequence whose 5th term is 48.**

3, 6, 12, 24, 48

$\frac{3}{16}$, $\frac{3}{4}$, 3, 12, 48

3. **Find the number of terms in 3, 6, 12, …, 384.**

$t_n = t_1 r^{n-1}$

$\frac{384}{3} = \frac{3(2)^{n-1}}{3}$

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$7 = n-1$

$\boxed{8 = n}$

$t_1 = 3$

$r = 2$

$n = ?$

$t_n = 384$

4. **In a geometric sequence, the first term is 4 and the third term is 324. Find t_2**

$t_n = t_1 r^{n-1}$

$324 = 4(r)^{3-1}$

$81 = r^2$

$\sqrt{81} = r$

$\pm 9 = r$

$r = \pm 9$

$t_2 = \pm 36$

$t_1 = 4$

$t_3 = 324$

$n = 3$

$r = ?$

4, −36, 324

5. In a geometric sequence, the second term is 28 and the fifth term is 1792. Determine the values of t_1 and r, and list the first three terms of the sequence.

$\times 4$ $\times 4$

$1, 28, 112, 448, 1792$

$t_1 = 28$

$n = 4$

$t_4 = 1792$

$t_n = t_1 r^{n-1}$

$1792 = 28 r^{4-1}$

$64 = r^3$

$4 = r$

6. In nature, bacteria reproduce by splitting in two so that one cell gives rise to 2, then 4, then 8, and so on. Suppose there were three bacteria originally present in a sample. Determine the general term that relates the number of the bacteria to the doubling period of the bacteria. State the values of t_1 and r in the geometric sequence formed.

$t_1 = 3 \quad r = 2$

$t_n = 3(2)^{n-1}$ General Term.