

Geometric Series

- A *geometric series* is the expression for the sum of the terms of a geometric sequence.
- The sum of a geometric series can be determined using the formula:

Sum \rightarrow

$$S_n = \frac{t_1(r^n - 1)}{r - 1}, r \neq 1$$

first term
number of term
common ratio

- A variation to the formula if the first term, common ratio, and n^{th} term are known is: (Used when n is unknown)

last term
 t_n

$$S_n = \frac{rt_n - t_1}{r - 1}, r \neq 1$$

- There is a special notation that is used to represent a series called Sigma Notation: Σ \leftarrow Sum of

For example, the geometric series $20 + 40 + 80 + 160$ has 4 terms, with the first term 20 and a common ratio of 2. The general term can be written as: $t_n = 20(2)^{n-1}$

$$t_n = t_1 r^{n-1}$$

The sum of these 4 terms can be written with sigma notation:

$$\sum_{k=1}^4 20(2)^{k-1}$$

$n \rightarrow 4$
 t_1
 r
term #1

integral values of k from 1 to 4

when $k = 1$

$$20(2)^0$$

$$20(1)$$

first term

\longleftrightarrow
represent the same series

$$\sum_{k=1}^4 10(2)^k$$

when $k = 1$

$$10(2)^1$$

20 - first

Examples:

1. Determine the sum of the first 8 terms of the following geometric series.

a) $5 + 15 + 45 + \dots$

$t_1 = 5$
 $n = 8$
 $r = 3$

$$S_8 = \frac{t_1(r^n - 1)}{r - 1}$$
$$= \frac{5(3^8 - 1)}{3 - 1}$$
$$= \frac{5(6560)}{2}$$

$S_8 = 16400$

2. Determine the sum of the following geometric series.

a) $\frac{1}{64} + \frac{1}{16} + \frac{1}{4} + \dots + 1024$

$t_1 = \frac{1}{64}$
 $t_n = 1024$
 $r = \frac{1}{4} \div \frac{1}{16}$
 $\frac{1}{4} \times \frac{16}{1} = \frac{16}{4}$
 $r = 4$

$$S_n = \frac{r t_n - t_1}{r - 1}$$
$$= \frac{4(1024) - \frac{1}{64}}{4 - 1}$$
$$= \frac{4096 - \frac{1}{64}}{3}$$
$$= \frac{262146}{64} \div 3$$

$S_n = \frac{87381}{64}$

3. For the geometric series:

$$\sum_{k=1}^n 3(2)^k$$

a) Write the first 4 terms of the series.

$k=1 \quad 3(2)^1 = 6$
 $k=2 \quad 3(2)^2 = 12$
 $k=3 \quad 3(2)^3 = 24$
 $k=4 \quad 3(2)^4 = 48$

First 4 terms:

$$6 + 12 + 24 + 48$$

b) Determine the sum of the series when the last term is 12288.

$$6 + 12 + 24 + 48 + \dots + 12288$$

$$t_1 = 6$$

$$r = 2$$

$$t_n = 12288$$

$$n = ?$$

$$S_n = \frac{r t_n - t_1}{r - 1}$$

$$= \frac{2(12288) - 6}{2 - 1}$$

$$= \frac{24570}{1} = 24570$$

Application

The format of a scrabble tournament is such that the losers of each round are eliminated from the next round. The winners continue to play until a final match determines the champion. If there are 512 participants, how many matches will be played?

$$t_1 = 256 \quad r = \frac{1}{2} \quad t_n = 1$$

$$\frac{512}{2} = 256 \text{ matches}$$

$$S_n = \frac{r t_n - t_1}{r - 1}$$

$$= \frac{\frac{1}{2}(1) - 256}{\frac{1}{2} - 1}$$

$$= \frac{-255 \frac{1}{2}}{-\frac{1}{2}}$$

$$S_n = 511 \text{ matches}$$