Transformations of Exponential Functions

Basic Function: $y = c^x$

Transformed Function: $f(x) = a(c)^{b(x-h)} + k$

- ***a*** **– vertical stretch. If negative a reflection in the x-axis**
- ***b*** **– horizontal stretch. If negative, a reflection in the y-axis.**
- ***h*** **– horizontal translation**
- ***k*** **– vertical translation**

Ex: Transform the graph of $y = 4^x$ **to sketch the graph of** $y = \frac{1}{2}(4)^{-2(x+5)} - 3$.

Describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts.

① State the parameters:

- y $a = \frac{1}{2}$ stretched vertically by a factor of $\frac{1}{2}$
- x $b = -2$ stretched horizontally by a factor of $\frac{1}{2}$ with a reflection in the y-axis
- x $h = -5$ moved left 5
- y $k = -3$ moved down 3

② Create a table of values

$y = 4^x$

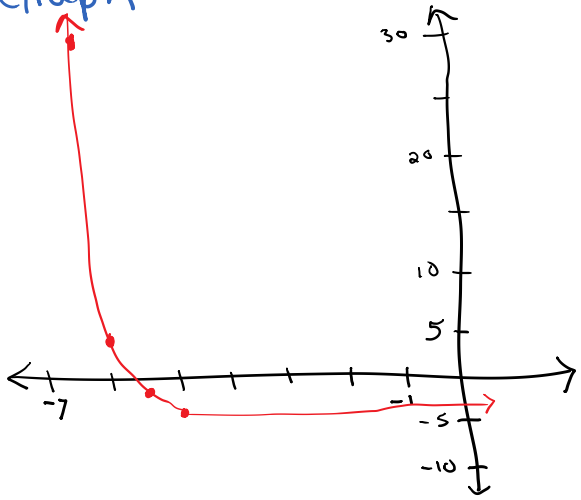
x	y
0	1
1	4
2	16
3	64

$y = \frac{1}{2}(4)^{-2(x+5)} - 3$

x	y
-5	$-2\frac{1}{2}$
$-5\frac{1}{2}$	-1
-6	5
$-6\frac{1}{2}$	29

mapping rule: $(x, y) \rightarrow \left(-\frac{x}{2} - 5, \frac{y}{2} - 3\right)$

③ Graph



d) Domain: $x \in \mathbb{R}$ Range: $y > -3$

Horizontal Asymptote: $y = -3$

y-intercept: approaching $y = -3$

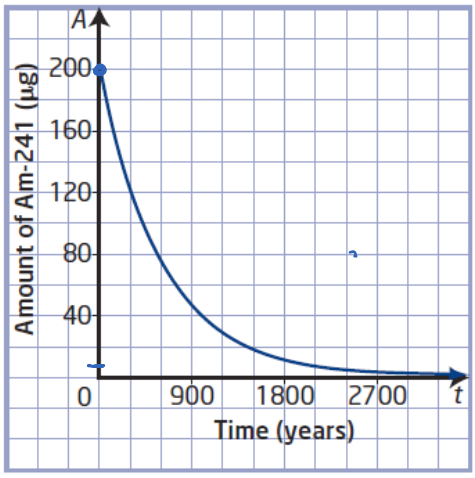
x-intercept: between -5.5 & -6

Ex: The radioactive element americium (Am) is used in household smoke detectors. Am-241 has a half-life of approximately 432 years. The average smoke detector contains 200 µg of Am-241.

↳ micrograms

a) **What is the transformed exponential function that models the graph showing the radioactive decay of 200 µg of AM-241?**

b) **Identify how each of the parameters of the function relates to the transformed graph.**



* need to understand the concept of "half-life"

↳ 200µg → (432 years) → 100µg → (432 years) → 50µg

a) need to write the equation of the function:

$y = c^x$

$y = \frac{1}{2}^{\frac{t}{432}}$

$y = 200\left(\frac{1}{2}\right)^{\frac{t}{432}}$

amt you started with.

* because it is a half-life the base (c) is $\frac{1}{2}$

* t in years

200µg is original amt

b) a = 200 vertical stretch of factor 200.

$b = \frac{1}{432}$ horizontal stretch

* No horizontal or vertical translations.