Exponential Equations

Write each expression as a power with a base of 2.

a) 4^3

$(2^2)^3$

2^6

b) $\frac{1}{8}$

$\frac{1}{2^3}$

2^{-3}

c) $8^{\frac{2}{3}}(\sqrt{16})^3$ → rewrite as an exponent

$(2^3)^{\frac{2}{3}}(16^{\frac{1}{2}})^3$

$2^{\frac{6}{3}}(16^{\frac{3}{2}})$

$2^2(2^4)^{\frac{3}{2}}$

$2^2(2^6) = 2^8$

Solve the equation: $4^{x+2} = 64^x$

* get a common base.

$4^{x+2} = (4^3)^x$

$4^{x+2} = 4^{3x}$

$\cancel{2}^{x} = \cancel{2}^{3}$

* solve for exponent

$x+2 = 3x$

$2 = 2x$

$1 = x$

Solve the equation: $4^{2x} = 8^{2x-3}$

$(2^2)^{2x} = (2^3)^{2x-3}$

$2^{4x} = 2^{6x-9}$

$4x = 6x - 9$

$-2x = -9$

$x = \frac{9}{2}$

Solve the equation: $9^{4x} = 27^{x-1}$

$(3^2)^{4x} = (3^3)^{x-1}$

$3^{8x} = 3^{3x-3}$

$8x = 3x - 3$

$5x = -3$

$x = -\frac{3}{5}$

Ex: If $1000 is invested in a term deposit that pay 8% per annum compounded semi-annually. How much will be in the account after 6 years.

1st 6 months: $1000 \times 0.04 = \$40$; $1000 + 40 = \$1040$

after 12 months: $1040 \times 0.04 = \$41.60$; $1040 + 41.60 = \$1081.60$

If we do it this way, it will be 24 calculations.

$1000 \times 1.04 = \$1040$ $\quad$ $1040 \times 1.04 = 1081.60$

↳ still would have to do 12 calculations

Has to be an easier way:

$1000 \times 1.04 \times 1.04 = 1081.60$

$1000 \times 1.04^2 = 1081.60$ (1 year)

$1000 \times 1.04^{12} = 1601.03$

$A = A_0 \chi^{\frac{t}{T}}$

* we can turn this into a formula.

A = final amt

A_0 = original amt

χ = growth decay factor

t = time interval in yrs

T = compounding period

Ex: After 10 years at 12% compounded quarterly Brian had $5000 in an account. What was the amount of the initial deposit? A_0

$A = A_0 \chi^{\frac{t}{T}}$

$\chi = 12\% \div 4$

$= 3\%$

↳ 1.03

$5000 = A_0 1.03^{\frac{10}{0.25}}$

$5000 = A_0 1.03^{40}$

$5000 = A_0 (3.262\ldots)$

$\$1532.78 = A_0$

Ex: A radioactive isotope has a half-life of 75 years. How long will it take for a 10 g sample to decay to 2 g?

$$A = A_0 \chi^{\frac{t}{T}}$$

$$\frac{2}{10} = \frac{10}{10}(0.5)^{\frac{t}{75}}$$

$$0.2 = 0.5^{\frac{t}{75}}$$

* we do not know how to solve algebraically yet. So we need to solve graphically.

$y_1 = 0.2$

$y_2 = 0.5^x$

graph & find point of intersection

2nd | trace | 5 Press enter, enter, enter $x = 2.32$

$\therefore 2.321 = \frac{t}{75}$ $t = 174.14$

Ex: The population of Smallville is increasing at a rate of 1.5% per year. If there are 100 000 people in Smallville today, how many people were there 5 years ago.

$$A = A_0 \chi^{\frac{t}{T}}$$

$$100\,000 = A_0(1.015)^{\frac{5}{1}}$$

$$\frac{100\,000}{1.07728\ldots} = \frac{A_0(1.07728\ldots)}{1.07728\ldots}$$

$$92826 = A_0$$